

Handwritten notes showing the derivation of the area of a square with a circular hole.

Area of Square  $x^2$

Area of Circle

$A = \pi r^2$

$A = \pi \left( \frac{10}{\pi} - \frac{2x}{\pi} \right)^2$

$A = \pi \left( \frac{100 - 40x + 4x^2}{\pi^2} \right)$

$A = \frac{100 - 40x + 4x^2}{\pi}$

$r = \frac{20 - 4x}{2\pi}$

$r = \frac{20}{2\pi} - \frac{4x}{2\pi}$

$r = \frac{10}{\pi} - \frac{2x}{\pi}$

SW

$\left( \frac{10}{\pi} - \frac{2x}{\pi} \right) \left( \frac{10}{\pi} - \frac{2x}{\pi} \right)$

$\frac{10}{\pi} \cdot \frac{10}{\pi} - \frac{10}{\pi} \cdot \frac{2x}{\pi} - \frac{2x}{\pi} \cdot \frac{10}{\pi} + \frac{2x}{\pi} \cdot \frac{2x}{\pi}$

$\frac{100}{\pi^2} - \frac{20x}{\pi^2} - \frac{20x}{\pi^2} + \frac{4x^2}{\pi^2}$

$\frac{100 - 40x + 4x^2}{\pi^2}$

$A_{\text{hole}} = x^2 + \frac{100 - 40x + 4x^2}{\pi}$

~~100 - 40x + 4x^2~~

~~100 - 40x + 4x^2~~

$$A = \pi r^2$$

$$A = \pi \left( \frac{10}{\pi} - \frac{2x}{\pi} \right)^2$$

$$A = \pi \left( \frac{100 - 40x + 4x^2}{\pi^2} \right)$$

$$A = \frac{100 - 40x + 4x^2}{\pi}$$

Area of  
Square  
 $x^2$

Area of  
Circle

$$r = \frac{20 - 4x}{2\pi}$$

$$r = \frac{20}{2\pi} - \frac{4x}{2\pi}$$

$$r = \frac{10}{\pi} - \frac{2x}{\pi}$$

SW

$$\left( \frac{10}{\pi} - \frac{2x}{\pi} \right) \left( \frac{10}{\pi} - \frac{2x}{\pi} \right)$$

$$\frac{10}{\pi} \cdot \frac{10}{\pi} - \frac{10}{\pi} \cdot \frac{2x}{\pi} - \frac{2x}{\pi} \cdot \frac{10}{\pi} + \frac{2x}{\pi} \cdot \frac{2x}{\pi}$$

$$\frac{100}{\pi^2} - \frac{20x}{\pi^2} - \frac{20x}{\pi^2} + \frac{4x^2}{\pi^2}$$

$$\frac{100 - 40x + 4x^2}{\pi^2}$$

~~Area of~~  
~~Circle~~

$$A_{\text{total}} = x^2 + \frac{100 - 40x + 4x^2}{\pi}$$

- you are all important to me  
- Dawnen Lacoste

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## 2.7 Analyzing Graphs of Functions and 2.7 Piecewise-Defined Functions

### • Test for Symmetry

Consider the equation in the variable  $x$  and  $y$ :

- The graph of the equation is symmetric with respect to the  $y$ -axis if substituting  $-x$  for  $x$  in the equation results in an equivalent equation

- The graph of the equation is symmetric with respect to the  $x$ -axis if substituting  $-y$  for  $y$  in the equation results in an equivalent equation

- The graph of the equation is symmetric with respect to the origin if substituting  $-x$  for  $x$  and  $-y$  for  $y$  in the equation results in an equivalent equation

→ for every point  $(x, y)$  on the graph, the point  $(x, -y)$  is also on the graph

→ for every point  $(x, y)$  on the graph, the point  $(-x, -y)$  is also on the graph.

→ for every point  $(x, y)$  on the graph, the point  $(-x, y)$  is also on the graph.

- In math, to "replace" something with something else can be tricky. to avoid making a mistake, ~~remember~~ remember CERS whenever you replace

Copy: Erase; Replace; Simplify.

## → Problems (Answer)

②  $y = -3x$

Test for x ( $y \rightarrow -y$ )

C:  $y = -3x$

E:  $( ) = -3x$

R:  $(-y) = -3x$

S:  $\frac{-y}{-1} = \frac{-3x}{-1}$

$y = 3x$

not equivalent  
NO  
x-axis  
symmetry

Test for y ( $x \rightarrow -x$ )

C:  $y = -3x$

E:  $y = -3( )$

R:  $y = -3(-x)$

S:  $y = 3x$

not equivalent  
NO  
y-axis  
symmetry

Test for origin  
( $x \rightarrow -x$ ;  $y \rightarrow -y$ )

C:  $y = -3x$

E:  $( ) = -3( )$

R:  $(-y) = -3(-x)$

S:  $\frac{-y}{-1} = \frac{3x}{-1}$

$y = -3x$

equivalent  
Yes  
Origin  
symmetry

③  $y^2 - x - 2 = 0$

Test for y ( $x \rightarrow -x$ )

C:  $y^2 - x - 2 = 0$

E:  $y^2 - ( ) - 2 = 0$

R:  $y^2 - (-x) - 2 = 0$

S:  $y^2 + x - 2 = 0$

no  
y-axis  
symmetry

Test for x ( $y \rightarrow -y$ )

C:  $y^2 - x - 2 = 0$

E:  $( )^2 - x - 2 = 0$

R:  $(-y)^2 - x - 2 = 0$

S:  $y^2 - x - 2 = 0$

Yes  
x-axis  
symmetry

Test for origin

C:  $y^2 - x - 2 = 0$

E:  $( )^2 - ( ) - 2 = 0$

R:  $(-y)^2 - (-x) - 2 = 0$

S:  $y^2 + x - 2 = 0$

No  
Origin  
symmetry